Handout 13
Compositional Semantics II:
Extensions & Intensions

PHILOSOPHY OF LANGUAGE

Let's learn a little more about semantic values by starting to build up a compositional semantics for a tiny fragment of English. Remember it matters what one's conception of propositional content is in this case since this determines (with compositionality) what your semantic values will look like.

We'll start with a possible worlds conception of propositional content since, in some ways, it is the simplest. In such a semantics, each word is associated with something called an "intension". Let's see what these look like.

Extensions and Intensions

(A) Predicates

The ***extension** of a predicate* is the set of things of which the predicate holds true.

E.g. "was a president" has an extension:

{George Washington, John Adams, Thomas Jefferson...}

These curly braces indicate we're talking about an entity called a *set*. You can think of this as just a collection or grouping. To say that an object O is in the set {…}, we write "O ∈ {...}". So, for example,

George Washington ∈ {George Washington, John Adams, Thomas Jefferson,...}

We can't always name the individuals in the extension of a predicate (sometimes there are infinitely many). So, sometimes we use the notation

{x : x was a president}

This means "the set of things x, such that x was a president". In our eventual semantics, extensions will be the 'interpretations' of predicates. The following notation

⟦was a president⟧

is read as "the interpretation of "was a president"". The following will thus be identical (in our framework).

⟦was a president⟧ = the extension of "was a president" =
{x : x is a president} = {George Washington, John Adams, Thomas Jefferson,...}

Important: in " ⟦was a president⟧ ", we" are talking about the *words* "was a president". In "{George Washington,...}" we are talking about the *person* George Washington.

As we saw, it is crucial on the truth-conditional conception of content that information concern not only how things *actually are* but how they *might have been*. Accordingly we will want to talk not only about what the extension of a predicate *is* but what it *would have been* in other circumstances.

Suppose we have a collection of possible worlds.

$W = \{w_1, w_2, w_3, ...\}$

In particular:

w_1 = the actual world
w_2 = like w_1 but Washington never went into politics, and Paine served as president for his terms
w_3 = like w_1 but Adams never went into politics, and Paine served as president for his terms

Then we can ask "what is the extension of "was a president"" at each of these worlds? Who were the past presidents in these worlds?

extension of "was a president" at w_1 =
{George Washington, John Adams, Thomas Jefferson,...}

extension of "was a president" at w_2 =
{Thomas Paine, John Adams, Thomas Jefferson,...}

extension of "was a president" at w_3 =
{George Washington, Thomas Paine, Thomas Jefferson,...}

So we can think of the words "was a politician" as determining for each possible world an extension (a set of individuals) at that world. A helpful way to think of this is by thinking of the word as determining a ***function***.

function: a mathematical abstraction which takes some number of *inputs* to a unique *output.*

In particular we can extend our notation as follows:

$[\![\text{was a president}]\!]^{\mathbf{w1}}$ = {George Washington, John Adams, Thomas Jefferson,...}
$[\![\text{was a president}]\!]^{\mathbf{w2}}$ = {Thomas Paine, John Adams, Thomas Jefferson,...}
$[\![\text{was a president}]\!]^{\mathbf{w3}}$ = {George Washington, Thomas Paine, Thomas Jefferson,...}

These are read "the extension of "was a president" at w_1 is ...". More generally we can think of the words "is a politician" as determining one extension for each possible world. We can note

$[\![\text{was a president}]\!]^{\mathbf{X}}$

You must have seen functions used with numbers. But functions can be over any objects. $[\![\text{was a president}]\!]^{\mathbf{X}}$ is a function over possible worlds: it takes as input a possible world and outputs an extension.

The ***intension*** *of a predicate*: a function from possible worlds to the extension of the predicate at those worlds.

NB: do not confuse *intensions* with intentionality, or intensional (contexts), or intentions (for action).

(B) Relations

Some predicates take more that one ‘argument’ to get a truth-value. For example “defeated” is word true of pair of objects rather than single objects. We can represent the extension of this word as a set of individuals, but a set of pairs of individuals (the pairs to which the application of the word yields a truth). So for example:

<John Adams , Thomas Jefferson>

would be in the extension but

<Thomas Jefferson, John Adams>

would not be. More generally we have

⟦defeated⟧ = {<George Washington, John Jay>, <John Adams, Thomas Jefferson>...}

And more abstractly:

⟦defeated⟧ = {<x,y>: x defeated y}

As before we can consider the *intension* of the relational predicate. Maybe in w_2, since Washington never went into politics <George Washington, John Jay> is not in the extension of “defeated” and instead

⟦defeated⟧$^{\mathbf{w2}}$ = {<Thomas Paine, John Jay>, <John Adams, Thomas Jefferson>, ...}

This is the extension (and interpretation) of “defeated” at w_2. More generally we have the intension:

⟦defeated⟧$^{\mathbf{X}}$

This is the *function* which takes a possible world as input and outputs the extension of “defeated” at that world.

(C) Names

The extension of referring extensions, like names, are normally taken to be their referents.

extension of a name: the referent of the name.

So in our notation we have

⟦George Washington⟧ = George Washington

Again, we want to know who this name picks out in other worlds. Let's suppose for now that the name always picks out the same person (we’ll discuss what this means much later). So we have

⟦George Washington⟧$^{\mathbf{w2}}$ = George Washington
⟦George Washington⟧$^{\mathbf{w3}}$ = George Washington

Etc.

So again we can think of the name "George Washington" as having an intension (a function from possible worlds to its extension). This is denoted:

$$[\![\text{George Washington}]\!]^{\mathbf{X}}$$

And since we are supposing that the name always picks out the same individual, this is a constant function.

$$[\![\text{George Washington}]\!]^{\mathbf{X}} = \text{George Washington}$$

(It's the function which, when it takes any world as input, outputs Barack Obama).

(D) Sentences

What's the extension of a sentence? Usually, by stipulation, a truth-value: *true* or *false*. So, e.g.

$$[\![\text{George Washington defeated John Jay}]\!] = \textit{true}$$

But

$$[\![\text{John Jay defeated George Washington}]\!] = \textit{false}$$

Just as the extension of predicates change with the world at which they're evaluated, so too a sentence can change its extension (truth-value) at a world of evaluation. Maybe for some world w_4, where Washington was embroiled in scandal

$$[\![\text{John Jay defeated George Washington}]\!]^{\mathbf{w4}} = \textit{true}$$

Again, we can think of each sentence as associated with an *intension:* a function from worlds to its extensions.

$$[\![\text{George Washington defeated John Jay}]\!]^{\mathbf{X}}$$

In this case this is a function from possible worlds to truth values. Think of this like a list:

$$\begin{bmatrix} w_1 & \textit{true} \\ w_2 & \textit{false} \\ w_3 & \textit{true} \\ \ldots & \ldots \end{bmatrix}$$

Note that this gives us the information about which worlds a sentence is true at. This is no coincidence: this information gives us a *set of worlds* at which the proposition is true. Or, if the possible worlds conception of a proposition is accurate, it *specifies the proposition* expressed by a sentence.