Handout 14
A “Toy” Compositional Semantic Theory

PHILOSOPHY OF LANGUAGE

"Toy" Compositional Semantics

Our syntax:

Lexicon of $\mathcal{L}$	
Nouns (N)	Sneha Ivan Raymond
Intransitive Verbs (V_I)	travels cooks
Transitive Verbs (V_T)	befriended

Syntactic Rule 1: A sentence S can only be made up of a noun N followed by a verb phrase VP.

Syntactic Rule 2: A verb phrase VP can be made up either
(i) of an intransitive verb V_I, or
(ii) of a transitive verb V_T followed by a noun N.

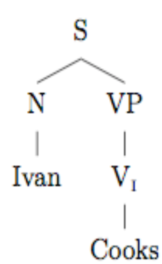
The semantics: Pair every word with an *intension* for that word. Recall that the brackets express our *interpretation (function)*: in our case, a function which takes an expression (and some parameters—in this case a world parameter) to an the expression's extension.

Semantics of $\mathcal{L}$				
Cat.	Lexical Item	Semantic Value of Lexical Item		
N	Sneha	$[\![\text{Senha}]\!]^x$	=	Sneha
	Ivan	$[\![\text{Ivan}]\!]^x$	=	Ivan
	Raymond	$[\![\text{Raymond}]\!]^x$	=	Raymond
V_I	travels	$[\![\text{travels}]\!]^x$	=	$\{a : a \text{ travels in } x\}$
	cooks	$[\![\text{cooks}]\!]^x$	=	$\{a : a \text{ cooks in } x\}$
V_T	befriended	$[\![\text{befriended}]\!]^x$	=	$\{\langle a, b\rangle : a \text{ befriended } b \text{ in } x\}$

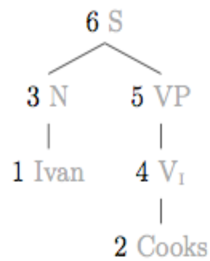
Claim: with these elements in place we can assign a meaning (a truth-conditional proposition) to every grammatically well-formed sentence of our language. To begin, take an example:

(1) Ivan Cooks.

Its syntactic structure:



Label each node for convenience:



Idea: at each stage of syntactic composition (each stage of "moving up the tree") we have a sentence-component whose semantic value should be specified by rules. We already know by the semantics for the lexicon:

$$\begin{aligned} [\![1]\!]^x &= [\![\text{Ivan}]\!]^x &&= \text{Ivan} \\ [\![2]\!]^x &= [\![\text{cooks}]\!]^x &&= \{a : a \text{ cooks in } x\} \end{aligned}$$

Semantic Rule 0: non-branching nodes have the same semantic values as their daughter nodes.

This rule alone gives us:

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$$\begin{aligned} [\![3]\!]^x = [\![1]\!]^x &= [\![\text{Ivan}]\!]^x &&= \text{Ivan} \\ [\![5]\!]^x = [\![4]\!]^x = [\![2]\!]^x &= [\![\text{cooks}]\!]^x &&= \{a : a \text{ cooks in } x\} \end{aligned}$$

We need a rule for branching nodes. In this case we want to "end up" with the following intension for the whole sentence: the function which takes a possible world *w* to *true* just in case Ivan cooks in *w*. The following rule will do the trick.

Semantic Rule 1: If a sentence S branches into a noun N and a verb phrase VP then then the semantic value of the sentence (i.e.$[\![S]\!]^x$) is the function *f* such that:
(i) $f(x)$ = *true* if $[\![N]\!]^x \in [\![VP]\!]^x$
(ii) $f(x)$ = *false* otherwise

To see this note that from the interpretation of the lexicon and Semantic Rule 0 we have:

$$[\![3]\!]^x = \text{Ivan}, \text{ and } [\![5]\!]^x = \{a : a \text{ cooks in } x\}$$

But Semantic Rule 1 tells us:

$[\![6]\!]^x$ is the function f such that

$$f(x) = \textit{true} \text{ if } [\![3]\!]^x \in [\![5]\!]^x \text{ and}$$
$$f(x) = \textit{false} \text{ if } [\![3]\!]^x \notin [\![5]\!]^x$$

Putting these two pieces of information together:

$$
\begin{aligned}
&[\![6]\!]^x \text{ is the function } f \text{ such that}\\
&\qquad f(x) = \mathit{true} \text{ if Ivan} \in \{a : a \text{ cooks in } x\} \text{ and}\\
&\qquad f(x) = \mathit{false} \text{ if Ivan} \notin \{a : a \text{ cooks in } x\}
\end{aligned}
$$

But a moment's reflection shows:

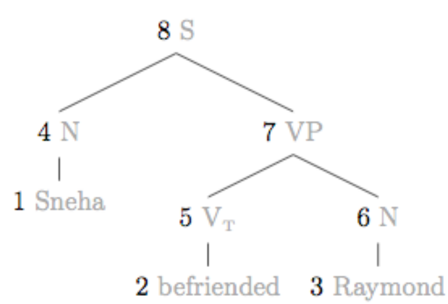
$$
\begin{aligned}
&[\![6]\!]^x \text{ is the function } f \text{ such that}\\
&\qquad f(x) = \mathit{true} \text{ if Ivan cooks in } x \text{ and}\\
&\qquad f(x) = \mathit{false} \text{ if Ivan doesn't cook in } x
\end{aligned}
$$

Why? What does it mean to say Ivan is in the set of persons *a* such that *a* cooks in x? It means that he is among the people who cooks in *x*. That is, he cooks in *x*. But *that's the very intension* that we *wanted* the whole sentence to have. So we're done.

Let's try another sentence.

(2) Sneha befriended Raymond.

Again, we can label the nodes for convenience:



From the interpretation of the lexicon and Semantic Rule 0 we have:

$$
\begin{aligned}
[\![4]\!]^x = [\![1]\!]^x = \quad & [\![\text{Sneha}]\!]^x && = \text{Sneha}\\
[\![5]\!]^x = [\![2]\!]^x = \quad & [\![\text{befriended}]\!]^x && = \{\langle a, b\rangle : a \text{ befriended } b \text{ in } x\}\\
[\![6]\!]^x = [\![3]\!]^x = \quad & [\![\text{Raymond}]\!]^x && = \text{Raymond}
\end{aligned}
$$

But we get stuck. Semantic rules don't tell us what the semantic value of the 7th node is. How can we figure it out?

Well, we know from Semantic Rule 1:

$$[\![8]\!]^x \text{ is the function } f \text{ such that}$$
$$f(x) = \mathit{true} \text{ if } [\![4]\!]^x \in [\![7]\!]^x \text{ and}$$
$$f(x) = \mathit{false} \text{ if } [\![4]\!]^x \notin [\![7]\!]^x$$

Which we know (from the interpretation of the lexicon) is:

$$[\![8]\!]^x \text{ is the function } f \text{ such that}$$
$$f(x) = \mathit{true} \text{ if Sneha} \in [\![7]\!]^x \text{ and}$$
$$f(x) = \mathit{false} \text{ if Sneha} \notin [\![7]\!]^x$$

We want the semantic value of 8 to be the function which takes a possible world *w* to *true* just in case Sneha befriended Raymond in *w*. So we want the semantic value of 7 to be the set of people who befriended Raymond in *w*. To get this we need this rule:

Semantic Rule 2: If a verb phrase VP branches into an intransitive verb V_T and a noun N, then:

$$[\![\mathrm{VP}]\!]^x = \{c : \langle c, [\![\mathrm{N}]\!]^x \rangle \in [\![\mathrm{V_T}]\!]^x\}$$

This looks complicated but it says something simple: when you put a noun and an intransitive verb together, the semantic value of the whole is a function from worlds to a set of things. Which set of things? The set of things such *c* such that *c* is paired with the extension of the noun in the extension of the verb at that world. In less convoluted talk: the set of *c* such that *c* performed the action specified by the verb to the individual specified by the noun (at the relevant world).

This gets things just right. By Semantic Rule 2:

$$[\![7]\!]^x = \{c : \langle c, [\![6]\!]^x \rangle \in [\![5]\!]^x\}$$

In other words (by what we already know):

$$[\![7]\!]^x = \{c : \langle c, \text{Raymond} \rangle \in \{\langle a, b \rangle : a \text{ befriended } b \text{ in } x\}\}$$

Which is just:

$$[\![7]\!]^x = \{c : c \text{ befriended Raymond in } x\}$$

Now recall that:

$$[\![8]\!]^x \text{ is the function } f \text{ such that}$$
$$f(x) = \mathit{true} \text{ if Sneha} \in [\![7]\!]^x \text{ and}$$
$$f(x) = \mathit{false} \text{ if Sneha} \notin [\![7]\!]^x$$

Now that we know what the semantic value of 7 is, we can plug that in:

$$[\![8]\!]^x \text{ is the function } f \text{ such that}$$

$$f(x) = \mathit{true} \text{ if Sneha} \in \{c : c \text{ befriended Raymond in } x\} \text{ and}$$
$$f(x) = \mathit{false} \text{ if Sneha} \notin \{c : c \text{ befriended Raymond in } x\}$$

Which, in other words is:

$$[\![8]\!]^x \text{ is the function } f \text{ such that}$$

$$f(x) = \mathit{true} \text{ if Sneha befriended Raymond in } x \text{ and}$$
$$f(x) = \mathit{false} \text{ if Sneha didn't befriend Raymond in } x$$

Let's try things one last time with a new example.

Lexicon and Semantic values

N	"Raymond"	Raymond
N_{pl}:	"bears" "snakes"	{a: a is a bear in x} {a: a is a snake in x}
A:	"skinny" "fat" "hairy" "gross"	{a: a is skinny in x} {a: a is fat in x} {a: a is hairy in x} {a: a is gross in x}
V_T:	"fears	{<a,b>: a fears b in x}

Syntactic Rules

S → NP - VP
VP → V_T - NP
NP → A - NP
NP → N
NP → N_{pl}

Semantic Rule 0: non-branching nodes have the same semantic values as their daughter nodes.

Semantic Rule 1: If a sentence S branches into a noun N and a verb phrase VP then then the semantic value of the sentence (i.e.$[\![S]\!]^x$) is the function *f* such that:

(i) f(x) = *true* if $[\![N]\!]^x \in [\![VP]\!]^x$
(ii) f(x) = *false* otherwise

Semantic Rule 2': If a verb phrase VP branches into an intransitive verb V_T and a noun phrase NP, then:

$$[\![VP]\!]^x = \{c: \langle c, [\![NP]\!]^x \rangle \in [\![V_T]\!]^x\}$$

Semantic Rule 2'': If a verb phrase VP branches into an intransitive verb V_T and a noun phrase NP, then:

$$[\![VP]\!]^x = \{c: \text{for every d in } [\![NP]\!]^x, \langle c, d\rangle \in [\![V_T]\!]^x\}$$

Semantic Rule 3: If a noun phrase NP branches into an adjective A and a noun phrase NP',

$$[\![NP]\!]^x = \{c: c \in [\![A]\!]^x \text{ and } c \in [\![NP']\!]^x\}$$

Last sentence: "Raymond fears fat hairy bears"

Triviality Worry: we just spent forever showing that the semantic value of "Raymond fears fat hairy bears" determines an *intension* which takes a possible world *w* to *true* just in case Raymond fears fat hairy bears in that world. But we knew that already! So the theory is abysmal: it generates trivial results but only after a highly non-trivial amount of work!

Mistake: the point of the theory is not to produce the outcomes that it does. In fact we often *test* the viability of the theory by using our *antecedent* knowledge of the outcomes.

Rather the theory is trying to *exhibit* the kinds of structural relations which compositionality would require. The theory thereby explains *the phenomenon of linguistic productivity:* how you can, through very limited exposure to a language, learn the meanings of an indefinite range of sentences you have never seen before.